OCR Maths FP1

Topic Questions from Papers

Proof by Induction

Answers

1	(iv)	B1		Explicit check for n = 1or n = 2
	$\mathbf{M}^k = \begin{pmatrix} 2^k & 3(2^k - 1) \\ 0 & 1 \end{pmatrix}$.	M1		Induction hypothesis that result is true for $\mathbf{M}^k$
		M1		Attempt to multiply $\mathbf{MM}^k$ or vice versa
	$\begin{pmatrix} 2^{k+1} & 3(2^{k+1} - 1) \\ 0 & 1 \end{pmatrix}$.	A1 A1		Element $3(2^{k+1} - 1)$ derived correctly All other elements correct
		A1	6	Explicit statement of induction conclusion
			12	

(Q9, June 2005)

2				
	$1^2 = \frac{1}{6} \times 1 \times 2 \times 3$	B1		Show result true for n = 1or 2
	$\frac{1}{6}n(n+1)(2n+1) + (n+1)^2$	M1		Add next term to given sum formula, any letter OK
		DM1		Attempt to factorise or expand and simplify
	$\frac{1}{6}(n+1)(n+2)\{2(n+1)+1\}$	A1	5	Correct expression obtained
		A1	**5**	Specific statement of induction conclusion, with no errors seen

(Q2, Jan 2006)

3	(i)	M1		Attempt at matrix multiplication
	$\mathbf{A}^2 = \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}$ $\mathbf{A}^3 = \begin{pmatrix} 8 & 0 \\ 0 & 1 \end{pmatrix}$	A1 A1	3	Correct $\mathbf{A}^2$ Correct $\mathbf{A}^3$
	(ii) $\mathbf{A}^n = \begin{pmatrix} 2^n & 0 \\ 0 & 1 \end{pmatrix}$	B1	1	Sensible conjecture made
	(iii)	B1 M1 A1 A1	 4	State that conjecture is true for n = 1 or 2 Attempt to multiply $\mathbf{A}^n$ and $\mathbf{A}$ or vice versa Obtain correct matrix Statement of induction conclusion
			$\boxed{8}$	

(Q7, June 2006)

4	(i)	B1		Correct expression for u_{n+1}
		M1		Attempt to expand and simplify
	$u_{n+1} - u_n = 2n + 4$	A1	3	Obtain given answer correctly
	(ii)	B1		State $u_1 = 4$ (or $u_2 = 10$)and is divisible by 2
		M1		State induction hypothesis true for
		M1		u_n
		A1		Attempt to use result in (ii)
		A1	5	Correct conclusion reached for u_{n+1}
			8	Clear,explicit statement of induction conclusion

(Q6, Jan 2007)

5	$(1^3 =)\frac{1}{4} \times 1^2 \times 2^2$	B1		Show result true for $n = 1$
	$\frac{1}{4}n^2(n+1)^2 + (n+1)^3$	M1		Add next term to given sum formula
		M1(indep)		Attempt to factorise and simplify
	$\frac{1}{4}(n+1)^2(n+2)^2$	A1		Correct expression obtained convincingly
		A1	5	Specific statement of induction conclusion
			5	

(Q2, June 2007)

6	(i)	M1		Obtain next terms
	$u_2 = 4$, $u_3 = 9$, $u_4 = 16$	A1	2	All terms correct
	(ii) $u_n = n^2$	B1	1	Sensible conjecture made
	(iii)	B1		State that conjecture is true for $n = 1$ or 2
		M1		Find u_{n+1} in terms of n
		A1		Obtain $(n + 1)^2$
		A1	4	Statement of Induction conclusion
			7	

(Q8, Jan 2008)

7	**B1**	Establish result is true, for $n = 1$ (or 2 or 3)
	M1	Attempt to multiply **A** and $\mathbf{A}^n$, or vice versa
	M1	Correct process for matrix multiplication
	A1	Obtain 3^{n+1}, 0 and 1
	A1	Obtain $\frac{1}{2}(3^{n+1} - 1)$
	A1	Statement of Induction conclusion, only if 5 marks earned, but may be in body of working
	$\boxed{6}$	

(Q4, June 2008)

8	**(i)** $13^n + 6^{n-1} + 13^{n+1} + 6^n$	B1		Correct expression seen
		M1		Attempt to factorise both terms in (i)
	(ii)	A1	3	Obtain correct expression
		B1		Check that result is true for $n = 1$ (or 2)
		B1		Recognise that (i) is divisible by 7
		B1		Deduce that u_{n+1} is divisible by 7
		B1	4	Clear statement of Induction conclusion
			7	

(Q7, Jan 2009)

9	i)	M1		Attempt to find next 2 terms
	$u_2 = 7$ $u_3 = 19$	A1		Obtain correct answers
		A1	3	Show given result correctly
	(ii)	M1		Expression involving a power of 3
	$u_n = 2(3^{n-1}) + 1$	A1	2	Obtain correct answer
	(iii)	B1ft		Verify result true when $n = 1$ or $n = 2$
	$u_{n+1} = 3(2(3^{n-1})+1) - 2$	M1		Expression for u_{n+1} using recurrence relation
		A1		Correct unsimplified answer
	$u_{n+1} = 2(3^n) + 1$	A1		Correct answer in correct form
		B1		Statement of induction conclusion
			5	
			10	

(Q10, June 2009)

10 (i)

$\mathbf{M}^2 = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix} \quad \mathbf{M}^3 = \begin{pmatrix} 1 & 6 \\ 0 & 1 \end{pmatrix}$

B1		Correct $\mathbf{M}^2$ seen
M1		Convincing attempt at matrix multiplication for $\mathbf{M}^3$
A1	**3**	Obtain correct answer

(ii) $\mathbf{M}^n = \begin{pmatrix} 1 & 2n \\ 0 & 1 \end{pmatrix}$

B1ft	**1**	State correct form, consistent with **(i)**

(iii)

M1		Correct attempt to multiply **M** & $\mathbf{M}^k$ or v.v.
A1		Obtain element $2(k + 1)$
A1		Clear statement of induction step, from correct working
B1	**4**	Clear statement of induction conclusion, following their working

(Q10, Jan 2010)

11

B1		Establish result true for n = 1or n = 2
M1		Add next term to given sum formula
M1		Attempt to factorise or expand and simplify to correct expression
A1		Correct expression obtained
A1	**5**	Specific statement of induction conclusion
	$\boxed{5}$	

(Q1, June 2010)

12

B1*		Establish result true for n = 1 or 2
M1*		Use given result in recurrence relation in a relevant way
A1*		Obtain $2^n + 1$ correctly
depA1	**4**	Specific statement of induction conclusion
$\boxed{4}$		

(Q3, Jan 2011)

13

B1		Establish result true for n = 1 or 2
M1*		Add next term to given sum formula
DM1		Combine with correct denominator
A1		Obtain correct expression convincingly
A1	**5**	Specific statement of induction conclusion, provided 1st 4 marks earned
$\boxed{5}$		

(Q2, June 2011)

14	(i)			M1 A1 A1 **[3]**	Attempt at matrix multiplication Obtain $\mathbf{M}^2$ correctly Obtain **given answer** correctly	
	(ii)		$\begin{pmatrix} 3^n & 0 \\ 3^n - 1 & 1 \end{pmatrix}$	B1 B1 **[2]**	3 elements correct 4th element correct	
	(iii)		$\begin{pmatrix} 3^{k+1} & 0 \\ 3^{k+1} - 1 & 1 \end{pmatrix}$	B1 M1 A1 B1 **[4]**	Show that their result is true for n = 1 or 2 Attempt to find $\mathbf{M}^k \cdot \mathbf{M}$ or vice versa Obtain correct answer Complete statement of induction conclusion	Must have 1st 3 marks

(Q7, Jan 2012)

15				B1 M1* DepM1 A1 B1 **[5]**	Verify result true when n = 1 Add next term in series Attempt to obtain 3^{k+1} correctly Show sufficient working to justify correct expression Clear statements of Induction processes, but 1st 4 marks must all be earned.	

(Q5, June 2012)

16	**(i)**		$\frac{2}{3}$, $\frac{2}{5}$, $\frac{2}{7}$	B1 B1 B1 **[3]**	B1 x 3, Obtain 3 correct values Justify **given** answer
	(ii)		$\frac{2}{2n-1}$	M1 A1 **[2]**	Fraction, in terms of n, with correct numerator or denominator Obtain correct answer a.e.f.
	(iii)		$\frac{2}{2(n+1)-1}$	B1ft M1 A1 A1 B1 **[5]**	Verify result true when n = 1, for their (ii), or n = 2, 3 or 4 Expression for u_{n+1} using recurrence relation in terms of n using their (ii) Correct unsimplified answer Correct answer in correct form Specific statement of induction conclusion, previous 4 marks must be earned, n=1 must be verified

(Q10, Jan 2013)

17			$2\left(2^{k+1}-2\right)+2$ or $2^{k+1}+2^{k+1}-2$	B1 M1 A1 A1 A1 B1 **[6]**	Establish result true for n = 1 or n = 2 Multiply **M** and $\mathbf{M}^k$, either order Obtain correct element Obtain other 3 correct elements Obtain $2^{k+2}-2$ convincingly Specific statement of induction conclusion, provided 5/5 earned so far and verified for n = 1

(Q4, June 2013)